Problems: Finite Automata

[Sipser/Chapter 2; HU/Chapters 2–3.]

- 1. Construct a DFA and a regular expression for the following languages over $\{0, 1\}$.
 - (a) $\{w \mid w \text{ has at least three 1s and at least 2 0s} \}$
 - (b) $\{w \mid w \text{ has even length and an odd number of } 1s\}$
 - (c) $\{w \mid w \text{ does not contain exactly two 1s}\}$
- 2. Show that the following language L over $\{0, 1\}$ is regular

 $L = \{w \mid w \text{ contains an equal number of occurrences of the substrings 01 and 10}\}.$

3. If A and B are regular languages over Σ , show that

$$PSHUFFLE(A, B) = \{ w = a_1 b_1 \dots a_k b_k \mid a_1 \dots a_k \in A \text{ and } b_1 \dots b_k \in B \}$$

is also regular.

4. If A and B are regular languages over $\{0, 1\}$, show that

 $A \xleftarrow{1} B = \{ w \in A \mid \text{ for some } y \in C, w \text{ and } y \text{ contain an equal number of } 1s \}$

5. Minimise the following DFAs.

