Indian Statistical Institute

Semester-II 2012-2013

M.Tech.(CS) - First Year

Class Test II (10 April, 2013)

Subject: Automata, Languages and Computation

Total: 20 marks

Solutions

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1. Fill in the blanks below with the correct expressions. [1×5=5]

   (a) Given a grammar $G = (V, T, P, S)$, a symbol $X \in V \cup T$ is said to be

   (i) **useful** if $S \stackrel{*}{\Rightarrow} \alpha X \beta \stackrel{*}{\Rightarrow} w$ for some $\alpha, \beta \in (V \cup T)^*$ and $w \in T^*$;

   You get $\frac{1}{2}$ if the 2 conditions are given separately; 0 if only 1 part is given.

   (ii) **nullable** if $X \stackrel{*}{\Rightarrow} \varepsilon$.

   Full credit was given for "$X \to \varepsilon$ or $X \to X_1 \ldots X_n$ and each X_i is nullable."

   (b) Given a grammar $G = (V, T, P, S)$, a production $A \to X_1 \ldots X_n$ is said to be a unit production if $n = 1$ and $X_1 \in V$.

   (c) Given a pushdown automaton (PDA) $P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$, the language accepted by P

   (i) by **final state** is given by $L(P) = \{w \mid (q_0, w, Z_0) \vdash_P^* (q, \varepsilon, \alpha),\ q \in F,\ \alpha \in \Gamma^*\}$;

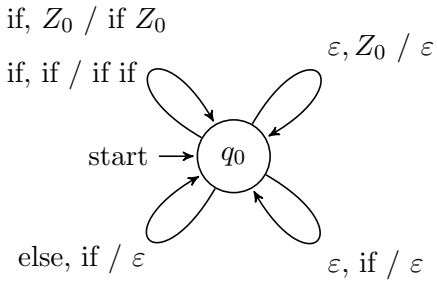
   (ii) by **empty stack** is given by $N(P) = \{w \mid (q_0, w, Z_0) \vdash_P^* (q, \varepsilon, \varepsilon),\ q \in Q\}$.

2. Recall that *postfix notation* is a method for writing arithmetic expressions in which every operator is written after all of its operands. For example, the postfix equivalent of $A \times B + C/D$ is $AB \times CD/+$. Write a context-free grammar (CFG) for arithmetic expressions in postfix notation involving variables and the operators $+$, $-$, $\times$ and $/$. You may assume that variable names consist of single letters only (as in the example above). You should use a single non-terminal S. [3]

   Answer: $S \to SS+ \mid SS- \mid SS\times \mid SS/ \mid A \mid \ldots \mid Z \mid a \mid \ldots \mid z$

P.T.O.

3. Let $L = \{w \mid w$ is obtained by taking a syntactically correct C program and removing everything other than the keywords `if` and `else` from it$\}$. Draw the state diagram of a PDA that accepts L by empty stack. You may assume that `if` and `else` are single symbols. [7]



-1 if the empty string is not handled correctly

-1 if excess `if`s are not popped at the end

4. Let $G = (V, T, P, S)$ be a CFG in Greibach Normal Form. Let $|V| = n$, $|T| = m$, $|P| = p$. Suppose that p_0 of the productions are of the form $A_0 \to aA_1A_2 \ldots A_k$ where k is a fixed number, $A_i \in V$ for $0 \leq i \leq k$ and $a \in T$. The remaining productions are of the form $A \to a$ where $a \in T$. Let $G' = (V', T, P', S)$ be a CFG in Chomsky Normal Form (CNF) obtained from G using the standard algorithm for conversion to CNF. Then:

$|V'| \leq \underline{\quad n + p_0(k-1) + \min(p_0, m) \quad}$ $|P'| \leq \underline{\quad p + p_0(k-1) + \min(p_0, m) \quad}$.

Your bounds should be tight. Briefly justify your answer. [5]

Full credit was given for $n + kp_0$ and $p + kp_0$.

For each calculation error (while calculating $|V'|$ or $|P'|$), you loose $\frac{1}{2}$.